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Dynamics of Orbits with Atmospheric Drag in Two Body Problem

Abstract

We investigate nature of the Lower Earth Orbit (LEO) under the atmospheric drag. As a result we impose decreasing nature of the satellite's altitude. We show that the satellite crashes into the Earth's orbit under a certain condition that if the satellite comes into the altitude of 145 km. We also investigate the velocity of the satellite moving around the Earth's atmosphere.

Keywords: Lower Earth Orbit (LEO), Atmospheric Drag, Two-Body Problem.

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Introduction

However there is no such existence of the two body problem in our real universe, in spite of that the study of two body problem plays a significant role in the study of Celestial Mechanics. The universe is a family of a huge numbers of bodies and everybody is related to another body by their mutual attraction forces, either that may be negligible or accountable. But if a problem can be considered such that it involves only a planet and its satellite, then it can be treated as a two body problem. Because the other effects due to the other body can be neglected in that case. This is how a two body problem can be established for the basic study of the celestial mechanics.

There a huge number of researches have been carried out in this _eld, which contains the dynamical model of the two body problem (Hough 1984; Seidelmann 1993; Breiter and Jackson 1998; Margheri et al. 2014). Few solutions also exist, either analytical, or in closed form, or may be numerical to the classical two-body problem. Similar problem with an additional forces due to the resistance of the medium surrounding by the attracting center has been discussed by Mittleman and Jezewski (1982), Mavraganis and Michalakis (1994), Brouwer and Hori (1961) etc,.

Mavraganis and Michalakis (1994) proposed a drag model, in which the resistance is proportional to the vector velocity and inversely proportional to the square of the mean distance, which gives a inhomogeneous second order linear di_erential equation. But we start with the atmospheric drag force as the resistance force, which is directly proportional to the square of the velocity of the satellite and exponentially with the radial distance.

Our concert is to develop the model for the lower earth orbit moving around the Earths atmosphere. However there are so many sources of perturbations affecting satellite orbital motion from the injection point until the ends of its lifetime. In general orbit perturbation can be divided into two different parts mainly, one is gravitational and another is non- gravitational. The gravitational are those due to oblates of the Earth, the zonal, the tesseral, and sectorial spherical harmonics and effect of the Suns or the Moons attraction. Again the non-gravitational perturbations are including atmospheric drag, solar radiation pressure and magnetic force, etc. Where the drag is dominating on the low Earth orbit, the solar radiation pressure is effective for the geosynchronous satellite and the magnetic force is due to the interaction of the Earth magnetic field with the dipole moment induced in the satellite. The gravitational potentials of the non-spherical earth models was initiated by Kozai (1959). Details of the gravitational potential theory and atmospheric drag force can be found in Chobotov (2002). The numerical simulation of the equation of the orbital motion was performed by Al-Bermani et al. (2012) and Metris and Exertier (1995).

Here our problem leads to a second order non-linear di_erential equation with a function of radial distance, time and velocity of the satellite. We have used Runge- Kutta method of numerical integration for numerical calculation. The atmospheric density is calculated as exponential functions of the satellite altitude from the Earths surface.

The paper is organized as follows: Sect. 2 gives the basic equation of the motion of the two body problem, Sect. 2.1 gives the dynamical model of the problem, Sect. 2.2 gives the model for the atmospheric drag, Sect. 3 gives the perturbation model and solvinng technique of the problem. In the Sect. 5 the graphical results are shown and in the last in Sect. 6 is about the conclusion part of the paper.

As a result we get the effect of atmospheric drag force decreases the lifetime of a satellite as well as it hampers the basic behavior of the satellite on its own orbit.

Equations of Motion

Let us consider an inertial system having its origin at O, in which the Newton's Laws of motion holds true. Let m_1 and m_2 be the masses of the two bodies, whose position vectors relative to O are r ~1 and r~2, respectively at a particular instant of time. Let ~R be the position vector of the center of mass of the pair. Also let ~r be the position vector of m_2 relative to m_1 . Thus we have

$$\vec{r} = \vec{r_2} - \vec{r_1},$$

where, r is the length of ~r. The equations of motion of the particles are given by

$$\ddot{\vec{r_1}} = \frac{Gm_2}{r^3}\vec{r},$$

$$\ddot{\vec{r_2}} = -\frac{Gm_1}{r^3}\vec{r}.$$

To make this analysis useful, let us fix the origin at m1, and subtracting the Eq. (1) from Eq. (2), we get

$$\ddot{\vec{r}} = \ddot{\vec{r_2}} - \ddot{\vec{r_1}} = -\frac{G(m_1+m_2)}{r^3}\vec{r}.$$

Thus the equation for the relative motion is

$$\ddot{\vec{r}} = -\mu\frac{\vec{r}}{r^3},$$

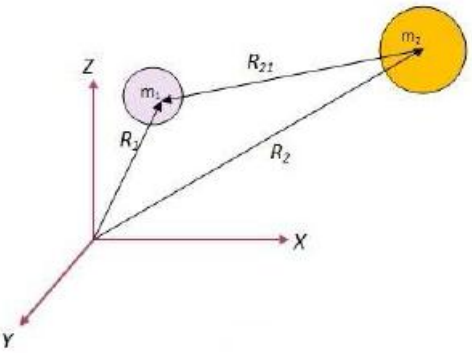

Fig. 1 Model of the two body problem

Where,

$$\mu = G(m_1 + m_2).$$

If one of the masses is negligible then the frame is inertial, otherwise the coordinate frame of relative motion is accelerated. Even so the equation of motion Eq. (4) has the same form as in inertial coordinate except that the mass of the central body is replaced by the totalmass of the system.

Dynamical Model of the Problem

Newtonian mechanics was initially developed in order to account for the motion of the planets in the solar system. The motion of a body object in the space is an integral part of the preliminary orbit determination process. The Kepler's problem subject in addition to a uniform force of constant magnitude and direction as given by Kozai (1959). Now Eq. 4 is written in terms of cartesian coordinate system as follows:

$$\ddot{x} = -\frac{\mu}{r^3}x, \qquad (6a)$$

$$\ddot{y} = -\frac{\mu}{r^3}y, \qquad (6b)$$

$$\ddot{z} = -\frac{\mu}{r^3}z, \qquad (6c)$$

where x; y; z are the coordinate axis of the cartesian coordinate system, and

Atmospheric Drag

When an object is not so far from the Earth's surface, it is subject to the drag due to the atmosphere. Actually the orbit perturbations are the cause of external torques acting about the center of mass of the satellite. When these torques arise from the space environment, specially from the atmosphere then it is called atmospheric drag.

The drag force is the resistance o_ered by Atmosphere to the satellite. This force always acts in the opposite direction to the motion of the satellite. The e_ect of the this force on a Lower Earth Orbit (LEO) decreases as the radial distance of the satellite increases. This is the cause that the force is maximum at the point of injection of the satellite and at the time of re-entry to the Earth surface. The action of drag on a satellite will cause it to spiral back into the atmosphere. Again as we are dealing with the LEO satellites, so we have to take into account the e_ect of the atmospheric angular motion, which we can take similar to the Earth's angular motion.

Now the acceleration experienced by the satellite due to atmospheric drag is computed using the following expression given by Kirk et al. (2013):

$$\vec{a_d}\left(\vec{r},\vec{v},t\right) = -\frac{1}{2}\rho\left(\vec{r},t\right)|\vec{v}_r|\vec{v}_r\frac{C_dA}{m}, \qquad (7)$$

V_r	=	Satellite velocity vector relative the atmosphere.
p	=	atmospheric density.
C_d	=	drag coefficient of the satellite.
A	=	reference area of the satellite.
m	=	mass of the satellite.

The drag coefficient is generally depends on the geometry of the body and is determined by experiments. The reduction in period due to atmospheric drag is given by (John, 1999).

$$\frac{dP}{dt} = -3\pi a\rho\left(\frac{C_dA}{m}\right), \qquad (8)$$

Atmospheric Density

The atmosphere of the Earth is surrounded by air and dust in a accountable sequence. The density of air at the Earth's surface is not equal to the density at a higher position from the Earth. We know that the behavior of the curve for density model is an exponential curve. It is seen that the density decreases as the altitude increases. The mathematical model of air density can be expressed as:

$$\rho(r) = 1.225 \times exp\left[-\left(\frac{r}{k_1} + \left(\frac{r}{k_2}\right)^{3/2}\right)\right], \quad (9)$$

where k_1 = 1:2 x 10^4m and k_2 = 2:2 _ 10^4m. Here *r* is the altitude of the position, where we want to calculate the air density.

Perturbations Model

This method is straightforward step by step integration of the two-body equation of motion with the perturbations. In addition to Eq.(4) the equation of motion can be taken as Al-Bermani et al. (2012)

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = \vec{a}_p, \quad (10)$$

where a_p is the sum of all perturbing acceleration to be included in the integration. The perturbing forces may be atmospheric drag force, radiation pressure, magnetic force, acceleration due to the Sun and the Moon etc. To numerically integrate this second order non-linear differential equation, we break Eq. (10) as follows:

$$\dot{\vec{r}} = \vec{v}, \quad (11)$$

and

$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_p. \quad (12)$$

Now these are pair of two first order differential equations. So the integration technique will be more easy.

Methodology to Solve the Problem

In general to solve a equations of motion of any dynamical model, there are so many different ways. The methods may be analytical, may be numerical, may be hypothetical etc. Here we deal with the problem of two bodies and used the numerical Runge-Kutta method of integration.

To solve numerically we have the Universal gravitational law, given by:

$$\vec{F}(r) = \ddot{\vec{r}} = -\frac{GMm}{r^3}\vec{r}. \quad (13)$$

Now, let us take two bodies, one with mass M and the other with mass m. The equivalent one-body problem can be reformed by defining the effective mass of the system as follows:

$$\mu = \frac{1}{\frac{1}{M} + \frac{1}{m}} = \frac{Mm}{(m+M)}, \quad (14)$$

where the force on this mass is given by the force between the two bodies. Now we discuss the dynamics of the LEO (Lower Earth Orbit), which is moving around the Earth under the attraction force of the Earth. So, here we take *M* as the Mass of the Earth and m the mass of the satellite; and suppose that the mass of the satellite be negligible compare to the mass of the Earth (m << M).

The above Eq.(11) and Eq.(12) can be written in terms of the cartesian X and Y coordinate axes as follows:

$$\dot{x} = v_x, \quad (15a)$$

$$\dot{v_x} = -\frac{\mu}{r^3}x + \vec{a_p}, \quad (15b)$$

$$\dot{v_y} = -\frac{\mu}{r^3}y + \vec{a_p}, \quad (15c)$$

$$\dot{y} = v_y, \quad (15d)$$

with $r = \sqrt{x^2 + y^2}$. The air density model is given by the e. (9)

To develop the program we set the initial conditions as follows:

$$x = r0,\ y = 0,\ vx = 0,\ vy = \sqrt{\frac{GM}{r0}}.$$

Environments

To get the solution of the problem we have used the C-code programing which includes the technique of numerical Runge-Kutta method of integration. It is a step by step method to get numerical data. We collect the data in a particular _le and then we plot the data as shown in the figures. The list of data, which we have collected are as follows:

1. Time step by ten seconds.
2. $\frac{1}{2\pi}\cos^{-1}(\frac{x}{r})$.
3. The radial distance covered by the satellite with corresponding time step.
4. The velocity of the satellite at every position when we recorded the radial distance.

The Graphical Results

After developing the code we check it in various conditions. Initially we consider the satellite is moving in a particular altitude from the Earth's surface. Then we show the affects of the atmospheric drag force on it, that how it affects the satellite, or what will may cause after a long time for the effect of the atmospheric drag forces.

Conventionally, we get a visible effects when we take the initial altitude of the satellite is about 200 Km. For collecting the data we use a too small time of step size to calculate the R-K method. We have taken the step size of 10 seconds to get more iterated data. Once we get the data we draw the graph by scattering plot in the MATLAB. Which are shown in the Figures 2, 3, and 4.

Effects of the atmospheric drag force on the satellite at altitude 200 Km is presented in figure 2 which shows that the altitude decreases with the time. It is also noticeable that initially it is decreasing with a slow rate, but as the time spends its rate of decreasing altitude increases. When the satellite is at nearly 135 Km, the altitude drops downward suddenly. As it is coming nearer to the Earth it is falling like a rock.

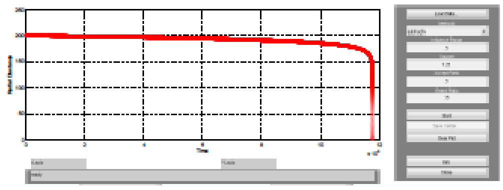

Fig. 2 Radial distance of the satellite with increasing time

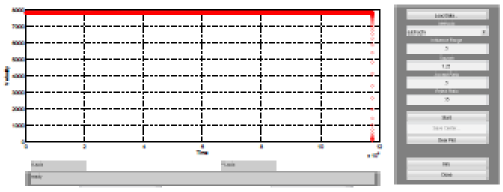

Fig. 3 Velocity of the satellite with increasing time

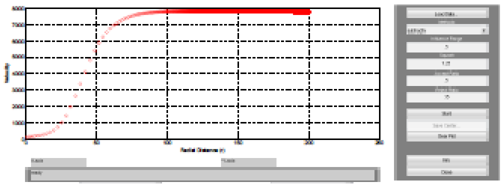

Fig. 4 The graph of radial distance vs velocity of the satellite

The effect of atmospheric drag on a LEO is also very important in case of discussing about the satellite's velocity. When the satellite comes into the interior atmosphere, due to the drag force the velocity decreases. It is seen in figure 3 that before falling down the velocity it increases for some time compare to the initial velocity. Again after falling down when it comes to very near to the atmosphere then the velocity is again effected as air density increases. The rate of changing velocity is also noticeable, which is clearly shown by the tail of the graph. It is also seen that when does the satellite crash into the Earth's surface. Figure 4 is presenting the radial distance Vs. velocity curve of the LEO satellite. It is visible that the velocity of the satellite is quite similar when it travels at the altitude level of 200 km to 100 km. But after that velocity goes down suddenly faster then previous one. This is due the high density of the Atmosphere at the Earth surface. It is seen that when the satellite crushes to the Earth surface then the satellite velocity is nearly about 110m/s^2.

Conclusions

In this paper we have shown the behavior of the LEO under the atmospheric drag. We have seen that the satellites moving under that force is being hampered compare to the satellites moving without this force. It is shown that if any satellite is moving around the altitude of 145 km then its will loss its position on the space and will crash into the earth surface in few times. It is also noticeable that the satellite will crash into the Earth's surface with a velocity of 110m/s. The nature of the velocity during its life under the altitude of 200 km has also imposed on this paper, which shows that satellite will change its velocity with respect to the time and the altitude.

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